Radius measurement and uncertainty evaluation using three-sphere reciprocity method

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Abstract. Three-dimensional (3D) measuring instrument has become widely applied by manufacturers due to the fast measurement capability and the flexible measuring procedure. It is also a crucial topic to evaluate or improve the measuring performance of these instruments, including the contact type like coordinate measuring machines (CMM) or the non-contact type like structure light scanners. Recently, the most commonly used standard feature is a sphere with a non-specular surface. It is because the ceramic materials is used to ensure the durability of the standard spheres, or is limited by the measuring principle of structure light scanners where the measured objects need to be lambertian surface. In the light of these limitations, the interferometry methods could not directly calibrate these standard spheres. In order to calibrate the radius and form of a non-specular sphere, this research focuses on the tactile method of spheres reciprocity method based on the previous researches. More details of the measuring principle and the formulas would be discussed. Final, the uncertainty of this method is evaluated using Monte Carlo method. The evaluated result of uncertainty is less than 10 nm for 15 mm radius sphere. This is very useful for common 3D measuring instrument calibration.

1 Introduction

Spheres are commonly used to be the artefact in order to provide the standard of sphere diameters or spacing values, for example the calibration sphere, ball-bar and ball plate[1, 2]. There are ISO 10360 international standards used the calibration sphere to evaluate the probing error of CMMs[3], also VDI 2634 standards using ball-bar artefact to evaluate and reverification the probing error and spacing error of optical scanners [4]. These standards all claim that these artefacts must be calibrated in order to ensure the dimensions is reliable.

There has been some techniques to calibrated the sphere radius, for example the direct interferometer method which is the most precise method used to calibrated dimension of silicon sphere[5]. But the limitation is that the sphere surface need to be specular, which is not suitable for calibrated the standard sphere for tactile CMM or optical CMS, because the standard spheres for tactile CMM need to be coated with hardness material to enhance the durability, which has the volume scattering effect, on the other hand, the optical CMS need to measure the object with lambertian surface. There are some methods to calibrate the non-specular surface sphere, for example the tactile probe or plate with the laser interferometer, but this method is hard to calibrate the whole sphere surface, instead the diameter at the equator is calibrated[6].

In this research, we focus on the tactile method to calibrated the sphere diameter with half sphere surfaces which is based on the method that former research[7] was applied to evaluate the 3D roundness of spheres. The method applied CMM and three spheres with unknown radius need to be touch against each other to format simultaneous equations. In this research, we will using Monte Carlo method to evaluate the measurement uncertainty of three-sphere reciprocity method which did not mention in the previous research.

2 Method

2.1 Principle of three-sphere reciprocity method

The three-sphere reciprocity method is performed by the CMM and the three spheres are touch with each other, the measuring procedure is shown in figure 1. In this method, sphere A is the sphere artefact, sphere B and sphere C are the ruby stylus ball of the CMM.

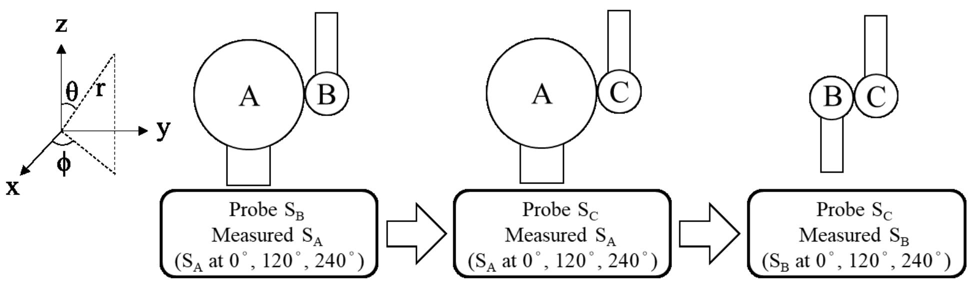

Fig. 1. Measuring procedure of three-sphere reciprocity method

The point cloud of coordinate of sphere surface is collect by the CMM, where the coordinate of sphere surface could be obtained. Then the nine simultaneous equations represent the center-to-center distance of two spheres are obtained as follow

$$M_1=R_A(\theta, \phi)+R_B(\theta, -\phi) \quad (1)$$

$$M_1'=R_A(\theta, \phi+120^\circ)+R_B(\theta, -\phi) \quad (2)$$

$$M_1''=R_A(\theta, \phi+240^\circ)+R_B(\theta, -\phi) \quad (3)$$

$$M_2=R_A(\theta, \phi)+R_C(\theta, -\phi) \quad (4)$$

$$M_2'=R_A(\theta, \phi+120^\circ)+R_C(\theta, -\phi) \quad (5)$$

$$M_2''=R_A(\theta, \phi+240^\circ)+R_C(\theta, -\phi) \quad (6)$$

$$M_3=R_B(\theta, \phi)+R_C(\theta, -\phi) \quad (7)$$

$$M_3'=R_B(\theta, \phi+120^\circ)+R_C(\theta, -\phi) \quad (8)$$

$$M_3''=R_B(\theta, \phi+240^\circ)+R_C(\theta, -\phi) \quad (9)$$

Then the equations could be rewritten as

$$M_0 = M_2 + M_3 - M_1 = 2\,R_C\,(\theta, -\phi) + [R_B\,(\theta, \phi) - R_B\,(\theta, -\phi)] \quad (10)$$

$$M_{120} = M_2' + M_3' - M_1' = 2\,R_C\,(\theta, -\phi) + [R_B\,(\theta, \phi+120^\circ) - R_B\,(\theta, -\phi)] \quad (11)$$

$$M_{240} = M_2'' + M_3'' - M_1'' = 2\,R_C\,(\theta, -\phi) + [R_B\,(\theta, \phi+240^\circ) - R_B\,(\theta, -\phi)] \quad (12)$$

Finally, the equations are simplified to

$$M_0 + M_{120} + M_{240} = 6\,R_C\,(\theta, -\phi) + \varepsilon \quad (13)$$

$$\varepsilon = [R_B\,(\theta, \phi) - R_B\,(\theta, -\phi)] + [R_B\,(\theta, \phi+120^\circ) - R_B\,(\theta, -\phi)] + [R_B\,(\theta, \phi+240^\circ) - R_B\,(\theta, -\phi)] \quad (14)$$

ε could be regarded as the form error of sphere B. The ε term is approximated to zero due to average out around the sphere by three divided, then the radius of sphere C at $(\theta, -\phi)$ direction can be approximated as

$$R_C\,(\theta, -\phi) = 1/6\,(M_0 + M_{120} + M_{240}) \quad (15)$$

Therefore, the radius of sphere A and sphere B at (θ, ϕ) direction can be computed as

$$R_A\,(\theta, \phi) = M_2 + R_C\,(\theta, -\phi) \quad (16)$$

$$R_B\,(\theta, \phi) = M_3 + R_C\,(\theta, -\phi) \quad (17)$$

The above derivation reveals that only one point to be calculate on sphere surface. More points on the sphere surface will be measured and using the same equations to solve the radius on half of the spheres.

2.2 Uncertainty evaluation

In this research Monte Carlo method is applied to evaluate the uncertainties of three-sphere reciprocity method. Here we list the terms of uncertainties due to CMM errors and some mathematical approximations.

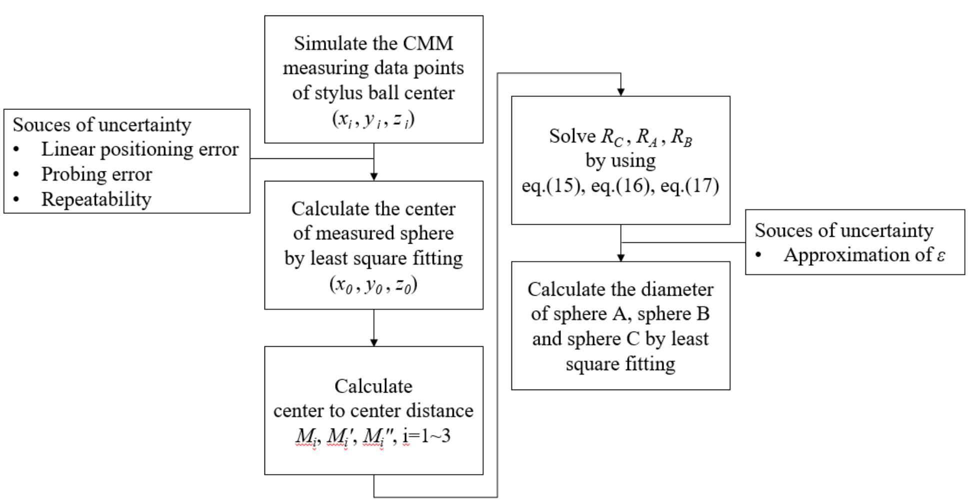

Fig. 2. Simulation procedure and uncertainties of three-sphere reciprocity method

2.2.1 Uncertainties of obtaining center-to-center distance

In this section, three major error sources that would affect the measured center-to-center distance was inlcuded.

- Linear positioning error of the CMM
- Probing error of the the CMM
- Repeatability of the CMM

The linear positioning error of the CMM is simulated by multiplying a scale factor (L_x, L_y, L_z) to (x_i, y_i, z_i) coordinate, therefore, the simulated data points (x_i', y_i', z_i') are

$$(x_i', y_i', z_i') = (L_x x_i, L_y y_i, L_z z_i) \quad (18)$$

The probing error of the CMM is simulated by using ellipsoid equation with parameters (a, b, c), the equation are shown as

$$(x_i'', y_i'', z_i'') = (a x_i' \sin\theta \cos\phi, b y_i' \sin\theta \sin\phi, c z_i' \cos\theta) \quad (19)$$

The repeatability of the CMM is conservatively assumed to be uniform distribution white noise (ε_x, ε_y, ε_z), hence, the data points with repeatability error (x''', y''', z''') are

$$(x_i''', y_i''', z_i''') = (x_i''+\varepsilon_x, y_i''+\varepsilon_y, z_i''+\varepsilon_z) \quad (20)$$

2.2.2 Uncertainty of ε approximation

By deriving the mathematical equations, we could find that the uncertainty of the zero approximation of ε is all contributed from R_B, which means that the probing error is mainly related to this uncertainty.

However, it might be averaged to zero by the rotational measurement and therefore obtain equation (15) although ε is actually an unknown variable in equation (13). But when using equation (15), the uncertainty of this approximation should still be considered. In this paper, we use the Monte Carlo technique to simulate the effect of non-zero ε.

3 Results and discussions

The nominal radius of the three spheres are set to R_A=15 mm, R_B=2.5 mm, R_C=2.5 mm. The parameters in the Monte Carlo simulation are shown in Table 1. The value of linear positioning error of the CMM is conservatively set to 5 µm per meter. The value of probing error of the CMM is set to 0.45 µm and the repeatability range is set to 0.27 µm, according to the Leitz Ultra CMM in our laboratory. Results below which combined the repeatibility are simulated with sample size equal to 50.

Table 1. Simulation parameters

Uncertainty term	**Value**
Linear positioning error	$L_x = L_y = L_z = 1.000005$
Probing error	$a = b = 1.000018$, $c = 1$ (nominal stylus ball radius = 2.5 mm)
Repeatability range	0.27 µm (uniform distribution)
ε approximation	Be calculated based on the value of other uncertainties (Fig. 3)

The histogram of ε for one loop of simulation is illustrated in Fig. 3. It appears that the assumption of zero-value of ε exist the residual errors which concentrate at 0 μm but spread out up to 1 μm. However, Fig. 4 shows that the deviation of R_A, R_B and R_C are all below 10 nm. The above results reveal that the three-sphere reciprocity method is non-sensitive to the error of the CMM and the residual error ε.

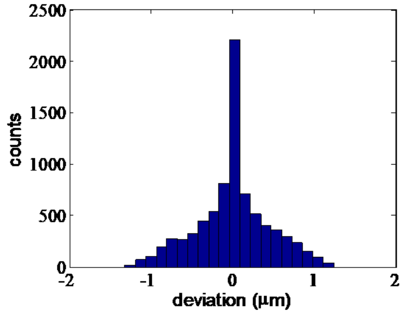

Fig. 3. Histogram of ε

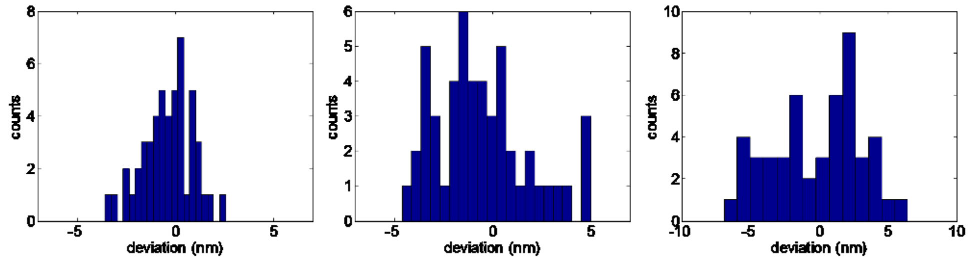

Fig. 4. Deviation of sphere radius (sample size = 50)

4 Conclusions

Previous results show that errors of the CMM contribute little uncertainty to the radius measurement of the sphere. And the results show that by using the CMM which specification is the same as Table 1, the uncertainty is expected to below 10 nm.

Future work would further complete the uncertainty evaluation, including the temperature effect, squareness of CMM and more detailed discussion of the form error of the measured sphere. The accuracy of form error of each sphere when using three-sphere reciprocity method would also be evaluated.

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