

A μ Kibble balance for direct realisation of small-scale masses and forces

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Abstract. Kibble balance experiments have allowed the kilogram to be redefined in terms of the Planck constant. Now that the redefinition is in place, the Kibble balance will allow SI traceable mass (or force) to be realised at any value and at any location. A feasibility study for a novel, scalable electrostatic ' μ Kibble balance', based on the National Physical Laboratory (NPL) next-generation Kibble balance system is presented and its expected performance calculated. A μ Kibble will allow *in-situ*, dynamic, small-scale (< g) mass measurements without the current precision limitation caused by subdivision of the kilogram. The measurements will be traceable through electrical calibration rather than transferable mass standards. The instrument will have a wide range of applications in both industry and research.

1. Introduction

On 20th May 2019 the kilogram was redefined. [1] The unit of mass is no longer determined by a physical artefact (the International Prototype Kilogram), but by a fixed value of the Planck constant, h . A test sample may be related to h either by the X-Ray Crystal Density technique (XRCD) or a Kibble balance.

Pre-redefinition, small masses could only be related from 1 kg *via* a series of subdivisions. At each stage of subdivision, the relative uncertainty is increased. At 1 mg, for example, the maximum permissible error (MPE) on an E₁ class weight is 0.3 % (300 times bigger than that for a 1 g weight). [2]

Redefinition has allowed mass to be directly realised at any value and at any location using an appropriately scaled instrument. This may allow measurements beyond current National Measurement Institute (NMI) precision to be realised in a research laboratory or in a production environment.

Precise small mass and force measurements are required for a variety of applications in fields including biotechnology development/production, environmental particulate monitoring, and atomic force microscopy.

In section 2 the concept of an electrostatic (ES) Kibble balance is briefly introduced, in section 3 the instrument subsystems are considered, and in section 4 the estimated performance of the system is presented.

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2. An electrostatic Kibble balance

2.1. The Kibble principle

The Kibble (formerly watt) balance was invented by Bryan Kibble at NPL in 1976. [3] It measures the mass of a specimen electrically, and so can relate the kilogram to quantum standards and the Planck constant.

The measurement process has two steps: firstly a ‘weighing’ step (where the weight, mg , of the mass is balanced by the force exerted by the current, I_B , through a solenoid in a magnetic field), and then a ‘moving’ step (where the solenoid is calibrated by moving the coil through the field at a measured velocity, v_B , and measuring the induced voltage, U_B). The results are then combined into the familiar Kibble equation, [4]

$$mgv_B = I_B U_B. \quad (1)$$

A similar relationship can be found if the electromagnetic (EM) solenoid actuator is replaced with an electrostatic actuator. Two parallel plates with a mutual capacitance C and a potential difference U , and which are separated by some distance z , can be characterised by a sensitivity constant k defined as follows.

$$k = \frac{dC}{dz} U \quad (2)$$

The electrostatic force, F , between the plates is then

$$F = \frac{1}{2} kU; \quad (3)$$

and the current, I , generated through the plates by moving one of the plates along z at a velocity v is

$$I = kv. \quad (4)$$

If it is assumed that the generated force is used to balance a weight, equations 3 and 4 may be combined to create an electrostatic equivalent to equation 1,

$$mgv = \frac{1}{2} IU. \quad (5)$$

Both an ES and EM balance require measurement of voltage, current and velocity; along with knowledge of local gravitational acceleration. Unlike an EM balance, an ES balance requires a stable and tuneable voltage source rather than a current source. The various subsystems required are described in more detail in section 3.

2.2. A next-generation balance

In 2014 Kibble and Robinson demonstrated that if the coil guidance mechanism is rectilinear and similar between both the weighing and moving mode on an EM Kibble balance, then the balance would be insensitive to the majority of misalignments. [5] Additionally, a single-axis (‘seismometer-style’) balance will be precise and easy to operate. [6] This analysis is equally applicable to an ES balance.

A single-axis electrostatic Kibble balance with a flexural linear guidance system is inherently easy to resize. There are many examples in the literature of ES actuators, sensors and flexures realised from the macro to the micro-electromechanical system (MEMS) scale (e.g. [7–11]). A balance can therefore be built to realise mass at small scales.

Furthermore, as a Kibble balance relates mass to electrical standards, calibration can be done *in-situ* through infrequent electrometer and voltmeter calibration (rather than difficult to handle mass standards) and is valid for time-varying signals.

The next sections briefly discuss the necessary subsystems of the ES balance, their estimated performance limitations and how this impacts the order-of-magnitude performance of an electrostatic Kibble balance over the mass range.

3. Balance subsystem considerations

3.1. Capacitor plates

The capacitor plates directly determine the sensitivity of the balance. For this analysis the capacitor plates are simply treated as two (or a series of) parallel plates of effective width w , which are separated a distance s by a dielectric with permittivity ϵ_d . This is illustrated in figure 3.1. One of the plates can translate in the z -axis with respect to the other by a maximum distance D .

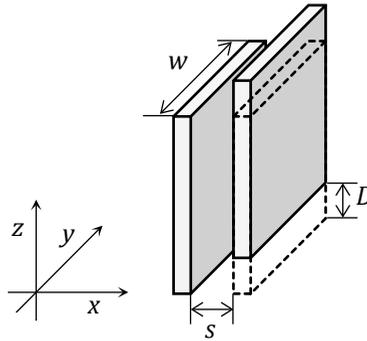


Figure 3.1. Simple schematic of assumed capacitor plate geometry. As illustrated, the left plate is fixed and the right plate is offset by the maximum displacement D .

It is assumed that the width of the plates will scale linearly with the characteristic size of the balance, r . *I.e.* $w = w_f r$, where w_f is some fixed geometry-dependent (and therefore optimisable) factor. It is further assumed that the separation can be no smaller than $10\ \mu\text{m}$ (to ease manufacture and assembly).

In this simple model and assuming the capacitor force is used to balance the weight of a mass, equations 2 and 3 can be used to derive a relationship between the mass scale and balance size.

$$m = \frac{\epsilon_d w_f U^2}{2gs} r \quad (6)$$

Similarly, equation 4 relates current and velocity.

$$I = \frac{\epsilon_d w_f U}{s} v \quad (7)$$

3.2. Voltage generation & measurement

A programmable voltage source is needed to balance the input force. As the system sensitivity is proportional to the voltage, it may be optimised to minimise overall uncertainty. For reasons including cost, voltage sources which can produce up to 1 kV will be used.

It is estimated that the voltmeter will be calibrated yearly (to minimise the associated time and costs). It is estimated that, under these conditions, an absolute uncertainty of 0.1 mV, and a relative uncertainty of 10^{-5} may be achieved.

3.1. Current measurement

Commercial electrometers, such as the Keysight B2980A, can measure small currents with a resolution 0.01 fA. It is therefore conservatively estimated that the electrometer will have an absolute uncertainty of 0.1 fA, and a relative uncertainty of 10^{-5} .

3.2. Velocity measurement

Average velocity can be determined, as in EM balances: by measuring displacement over a measured time.

NMI-level optical interferometry can be used to make traceable displacement measurements accurate to 10 pm. [12] It is estimated that a low-cost interferometer with reasonable alignment insensitivity (some of which are available off-the-shelf) will have an absolute uncertainty of 1 nm, and a relative uncertainty of 10^{-6} .

The integrating time of the system may be optimised and is assumed to be known with an uncertainty of 10 μ s, and a relative uncertainty of 10^{-6} , based on the trigger specifications of the electrometer.

3.3. Linear guidance mechanism

A flexural linear guidance mechanism can offer repeatable, precise motion at any scale. It must be maximally compliant in one axis, to maximise sensitivity in weighing mode; and minimally compliant in the other five (rotational and translational) axes, to minimise misalignment between the weighing and moving mode.

The maximum displacement possible in moving mode will be limited by both: the maximum permissible stress on the flexures, and the ability of the driving actuator to overcome the stiffness of the flexures.

If geometric linearity is assumed, the maximum flexure stress when deformed will scale linearly with the characteristic length of the flexure, with a proportionality constant given by the specific flexure geometry. To avoid plastic deformation, stress should be kept below 25 % of the material yield stress. [13] For a trial flexure geometry, the stress-limited maximum displacement is $D_{\sigma} = 0.1r$.

The carriage displacement, d , is modelled assuming critically damped simple harmonic motion,

$$M\ddot{d} = F - Cd - 2\sqrt{MC}\dot{d}. \quad (8)$$

Where the total carriage mass is $M = 10m$ (*i.e.* linearly proportional to the test mass) and flexure stiffness is estimated to be $C = r \cdot 100 \text{ Nm}^{-2}$, based again, on FEM modelling and assuming linearity. The stiffness-limited maximum displacement, D_C , is the amplitude of d , and is not dependent on the mass scale.

The combined maximal allowable displacement D of the carriage during moving mode is then simply,

$$D = \min(D_{\sigma}, D_C). \quad (9)$$

It is estimated that, due to the finite off-axis compliance of the linear guidance mechanism, there will be some misalignment between the weighing and moving mode, but this should introduce a relative uncertainty of no more than 10^{-5} .

3.1. Gravity modelling

To obviate the need to measure local absolute gravity at the location of the balance, it is assumed that g will be modelled rather than measured. A reasonable, conservative uncertainty on this value is in the order of 10^{-5} (~10 mGal). [14]

4. Estimated balance performance

4.1. Model equation

Equation 5 may be used to create an approximate model equation for mass measurement by an ES balance.

$$m = \frac{\alpha U I t}{4\pi g D} \quad (10)$$

Two simplifying assumptions have been made.

1. There is some misalignment between the moving and weighing mode such that $I = \alpha k v$ and $\alpha \approx 1$.
2. The velocity in moving mode is equivalent to $v = 2\pi D/t$, where D is the maximum distance moved along z and t is the sampling time.

In accordance with the GUM, [15] if the variables are considered uncorrelated, then the uncertainty on the measured mass is, to the first order, as below.

$$\frac{u^2(m)}{m^2} = \frac{u^2(\alpha)}{\alpha^2} + \frac{u^2(U)}{U^2} + \frac{u^2(I)}{I^2} + \frac{u^2(g)}{g^2} + \frac{u^2(D)}{D^2} + \frac{u^2(t)}{t^2} \quad (11)$$

Where $u(x)$ is defined as the standard uncertainty on a quantity x .

The value of the input parameters to the model equation are given along with their uncertainties in table 1.

Table 1. Input parameters to the model equation and their assumed $k = 1$ uncertainty.

| Parameter | Symbol | Value | Uncertainty | Notes |
|----------------------------|--------------|---|------------------------------|--------------------|
| Mass | m | 1 ng to 1 g | $u(m)$ | Region of interest |
| Dielectric permittivity | ϵ_d | 8.85×10^{-12} F/m | 0 | |
| Mode alignment | α | 1 | $10^{-5} \alpha$ | Fixed |
| Gravitational acceleration | g | 9.811 m/s ² | $10^{-5} g$ | |
| Capacitor voltage | U | 10 mV to 1 kV | $0.1 \text{ mV} + 10^{-5} V$ | |
| Capacitor geometry factor | w_f/s | 1 m^{-1} to 1 nm^{-1} | – | Optimised |
| Integration time | t | 0.1 s to 10 ks | $10 \mu\text{s} + 10^{-6} t$ | |
| Induced current | I | $\frac{2\pi\epsilon_d w_f U d}{st}$ | $0.1 \text{ fA} + 10^{-5} I$ | Calculated |
| Maximum displacement | D | $\min(D_\sigma, D_C)$ | $1 \text{ nm} + 10^{-6} d$ | |

4.2. Optimised performance

The system variables (related to capacitor voltage, plate geometry and sampling time) have been pareto optimised to minimise the relative uncertainty across the mass range. The values of these variables are plotted in figure 4.1. At the smallest mass value considered (1 ng), the

capacitor geometry factor has been maximised (to maximise the capacitor sensitivity) and integration time minimised (to maximise velocity). The optimised capacitor voltage also decreases with mass value. This reduces the current produced (which decreases linearly with voltage), but also decreases the force, and therefore mass value, by a larger amount (as it decreases as the voltage squared).

The characteristic length of the balance, r , in metres varies as approximately the square root of the mass value, m , in kilograms. *I.e.* a balance with a length scale of 1 μm is required to measure 1 ng, and a balance of 32 mm can measure 1 g.

The sensitivity of the overall uncertainty to the input parameter uncertainty is plotted in figure 4.2. With optimised parameters, the mass measurement uncertainty is similarly sensitive to current and voltage measurement.

Figure 4.3 shows the estimated combined uncertainty on mass measurement of the electrostatic balance. The dominant uncertainty source below 10 ng is the current measurement. Above 1 mg the uncertainty in alignment, voltage, current and gravity all contribute similar levels. In between these extremes (the majority of the region of interest) displacement measurement is the dominant uncertainty contributor.

Also plotted in figure 4.3 is the relative uncertainty of E₁ class masses. The expected uncertainty of an ES Kibble balance outperforms E₁ class weights at 100 mg and below. A balance will continue to offer useful measurements down to the ng level.

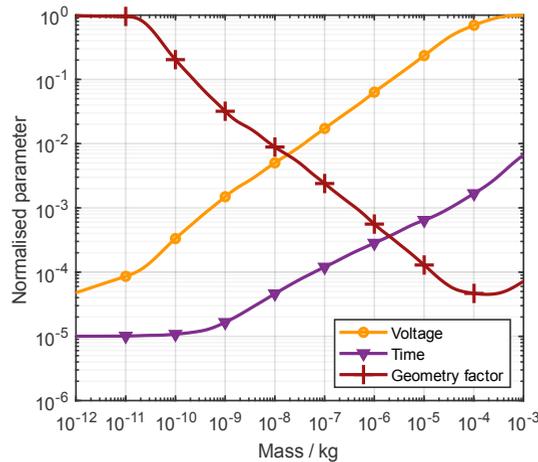


Figure 4.1. Value of the pareto optimised parameters over the mass range. Plotted in yellow circles is voltage (U), in purple triangles is the integration time (t), and in red crosses is the capacitor geometry factor (w_f/s). All parameters are normalised, plotted as a ratio of the variable to its upper bound (as given in table 1).

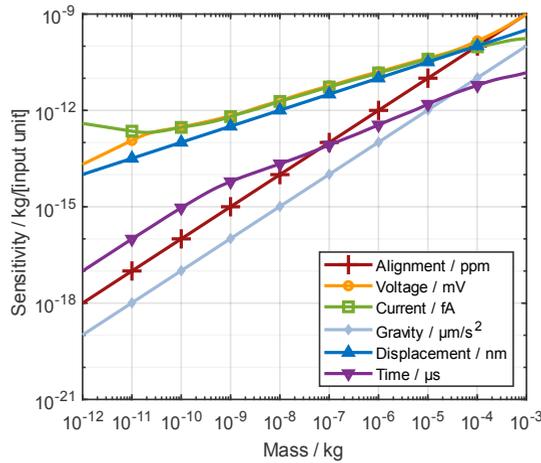


Figure 4.2. Sensitivity of total mass measurement uncertainty to the contribution parameter uncertainty (when expressed in the labelled units).

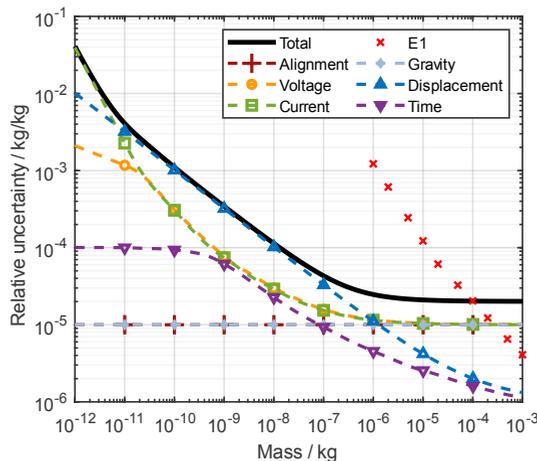


Figure 4.3. Optimised relative uncertainty ($k = 1$) of mass measured by an electrostatic Kibble balance over the measured mass. Shown in *solid black* is the total uncertainty. The *dashed* lines show the various contributors as labelled. The *red crosses* show the equivalent uncertainty (assuming a triangular distribution) for E₁ class weights.

5. Conclusion & future work

A simple model of a single-axis electrostatic Kibble balance has been presented and some design variables have been optimised to estimate the minimal achievable measurement uncertainty. It has been shown that between 1 ng and 100 mg the balance may be more precise than existing traditional mass standards.

The balance also offers traceable *in-situ* measurement of time-varying masses and forces calibrated using electrical standards, a benefit which may be utilised across the mass scale.

Work needs to be done to both build a proof-of-concept balance, and to consider the steps required to integrate such a balance into a measurement chain.

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