Improvement of the accuracy of the logometric impedance meters in wide frequency range
Amélioration de la precision de mesure des mesureurs logométriques de l'impédance sur une large band de fréquences

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Abstract Logometric impedance meter with variational correction of the uncertainty, caused by finite loop amplification and admittance between protecting amplifier inputs has been developed. To correct named uncertainty, the variation of the protecting amplifier gain is provided and additional measurement of the voltage drop on the standard is implemented. Appropriate calculation eliminates influence of the described uncertainty source on the result of measurement. Proposed approach has been used in the logometric impedance meter MNS1200, delivered to Siberian Institute of Metrology to be used in secondary standard of inductance for frequency up to 1 MHz.

1. Introduction
To measure impedance parameters first investigators used balanced bridge devises or movable-pointer indicators [1, 2]. Last type of measurement instruments was very simple but had rather big errors, because of the result of measurement depended on the applied voltage (current). This situation sharply changes from the beginning of seventies, when the first integral Operational Amplifiers (OpAmp) and Microprocessors were developed. At this time the movable-pointer indicators have been performed into the new class of measuring devises – auto-balancing or logometric meters [3, 4, 5, 6, 7]. Today these meters have uncertainty better than 0.01%, and operate on frequency range from DC to tens MHz. Many companies manufacture now such devises (HP, Agilent, TeGam, IetLab, Wine Kerr, etc).

2. Main Idea
First automatic logometric meters [8] used an OpAmp, with feedback, created by measured and standard impedances \( Z_x \) and \( Z_o \). Input and output voltages of this OpAmp were measured by simple vector voltmeter, their ratio was determined. The ratio of these voltages is approximately equal to the ratio of the impedances. But the voltage ratio in this case depends on the loop gain of the amplifier, so that appropriate uncertainty \( \delta_Z \) of measurement is given by formula:

\[
\delta_Z \approx \left( Z_0 + Z_x \right)/KZ_x.
\]  (1)

This uncertainty can achieve significant values on the high frequencies range, where the amplifier gain \( K \) sharply decrease. Other source of uncertainty here is caused by difficulties of the four terminal connections of both \( Z_x \) and \( Z_o \) on low impedance measurement range. To eliminate influence of these sources of uncertainty, H.P. Hall proposed the auto-balancing bridge [9], where voltage drops on the measured and standard impedances \( Z_x \) and \( Z_o \) are measured by the differential vector voltmeter. In this case amplifier gain \( K \) theoretically doesn’t influence the result of measurement and OpAmp protects measuring circuit only.

Real measuring circuit contains not only the measured and standard impedances \( Z_x \) and \( Z_o \), but some impedance \( Z_g \) connected between negative and positive inputs of the protecting OpAmp. Usually admittance \( 1/Z_g \) consists of the cable admittance and the admittances of the low potential ports of the measured and standard

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impedances $Z_x$ and $Z_o$. Uncertainty caused by this impedance, can be estimated by the formula:

$$\delta_z \approx \frac{Z_o}{KZ_g}$$  \hspace{1cm} (2)

On high impedance ranges of measurement (when $Z_o$ increase, so that $Z_o > Z_g$) and on high frequency range (where $K$ is low) this uncertainty can achieve considerable values. Today manufactures propose to enter into the devise the value of $Z_g$. Process of these data can eliminate or decrease this uncertainty [10, 11,12, 13]. Of course, amplifier gain and $Z_g$ aren’t stable and can change in rather wide range. In this way such correction can’t eliminate fully described uncertainty.

In this report we describe the new approach, which eliminate fully described above uncertainty. This approach is based on the variational method of uncertainty correction [14]. In compliance with this method to eliminate any uncertainty we need to vary the influence of the appropriate source on the result of measurement in well known ratio or on well known value. We determine twice the measured value: before and after variation. After that we solve the system of two equations, which describes these measurements, eliminate the uncertainty and get accurate results. In case if the influence of N uncertainty sources has to be eliminated, the N appropriate variations has to be done and the system of N+1 equations has to be solved to get accurate result of measurement.

The circuit diagram of the logometric impedance meter with variational correction of the described uncertainty is shown on Fig. 1. Logometric meter consists of the generator G and protecting amplifier A, which have the gain $K$. The measured and the standard impedances $Z_x$ and $Z_o$ create the feedback loop of the

![Fig 1. Circuit diagram.](image)

The switcher K connects the differential inputs of the vector voltmeter VV to the measured and the standard impedances $Z_x$ and $Z_o$, so that appropriate voltage drops are measured. Microcontroller MC processes results of voltage measurements and display the impedance parameters on the Display.

In ideal case, if the amplifier A gain $K$ is infinite, the input voltage of the protecting amplifier is equal to zero and no current flow through impedance $Z_g$. If the amplifier A gain $K$ is finite, its input voltage isn’t equal to zero, so that the part of current which flow through $Z_g$ creates appropriate uncertainty (effect of quazy increasing $Z_g$).

Let, in compliance with the variational method, we will vary the amplifier A gain $K$ on the well known ratio $K_v$. To this end the switched divider $K_v$ is connected serially with the amplifier A. This divider has transfer coefficient 1 or $K_v$, controlled by the microcontroller MC. To eliminate described uncertainty, the vector voltmeter, under the control of the MC, provide three measurements:

- measurement of the voltage drop $U_x$ on the $Z_x$;
- measurement of the voltage drop $U_0$ on the $Z_o$ when divider $K_v$ has a transfer coefficient equal to 1;
- measurement of the voltage drop $U_{0v}$ on the $Z_o$ when divider $K_v$ has a transfer coefficient equal to $K_v$.

These three measurements are described by the system of three equations:

$$U_x = I_x Z_x$$

$$U_o \left(1 - \frac{1}{K_v Z_g}\right) = I_x Z_o$$  \hspace{1cm} (3)

$$U_{0v} \left(1 - \frac{1}{K_v K + Z_g}\right) = I_x Z_o$$

MC solves digitally the system of equations (3) and finds the accurate ratio $Z_x/Z_o$. For further analysis there is better to get analytical solution. Taking into account that relative difference $\delta U$ between $U_o$ and $U_{0v}$ is very small and depends on the uncertainty itself, we get the next approximate formula for precised calculation of the measured value $Z_x$:

$$Z_x = Z_o \frac{U_x}{U_0} \left(1 + \frac{K_v}{1 - K_v} \frac{\delta U}{U_0} \right).$$  \hspace{1cm} (4)

where: $\delta U = U_0 / U_{0v} - 1$

Uncertainty of the proposed meter doesn’t depend at all on amplification of the protecting amplifier A on all the range of measurement and in frequency range. In this case its uncertainty budget includes, as usual logometric impedance meter, nonlinearity and sensitivity of the vector voltmeter, short term stability of the generator voltage magnitude, standard’s uncertainty, etc., but adds uncertainty of the variation and greater influence of the
vector voltmeter sensitivity due to the increased quantity of voltage measurements.

Let us estimate the increasing in uncertainty caused by last factors.

Vector voltmeter measures two quadrature components \(a\) and \(b\) of the input signals \(U_x\) and \(U_o\) in the known logometric meter and components of three input signals \(U_x\), \(U_0\) and \(U_{ov}\) in proposed meter. Its transfer function, which takes into account additive error \(\Delta\) (the limited sensitivity, caused by the noise) can be described by two functions:

\[
a = kU_x + \Delta; \quad b = kU_o + \Delta
\]

(5)

here \(k\) is vector voltmeter gain and \(U_x\) and \(U_o\) are quadrature components of the measured signals.

Let the vector voltmeter dynamic range is \(A_m\), and it is used in such way that:

\[
A_m = a^2 + b^2, \quad A_m \geq a_r^2 + b_r^2
\]

(6)

Known formulas for impedance component calculation [3], formulas (5) and (6) we get the expressions of the dependence between the uncertainty of the measured impedance components and the vector voltmeter additive conventional uncertainty \(\delta\) in the known logometric meter, described in [9]:

\[
\delta_{r(b)} \approx -\delta; \quad \Delta_{r(b)} / R_0 \approx \delta; \quad \Delta_{b(r)} / R_0 \approx \delta.
\]

(7)

where \(\delta_{r(b)}\) is the multiplicative uncertainty of the active (reactive) impedance component measurement and \(\Delta_{r(b)}, \Delta_{b(r)}\) are the additive components of the uncertainty of measurement when main active(reactive) component of impedance is measured.

Using formulas (4), (5) and (6) we get the expressions for the dependence between the uncertainty of the the measured impedance components \(\Delta(\delta_{r(b)})\) and the vector voltmeter additive conventional uncertainty \(\delta\) in proposed impedance meter:

\[
\Delta(\delta_{r(b)}) = \sqrt{2} \delta / \frac{K_e}{1-K_e}
\]

(8)

Formulas (8) show that the proposed algorithm of measurement only slightly increase the influence of the vector voltmeter noise on the uncertainty of the result of measurement.

The proposed algorithm needs three measurements instead on two measurement in the known meters, i.e. decreases the speed of measurement on one third. Because of it, this algorithm is preferable to be used on high frequency and high impedance parts of the range of measurement only. In such way we will decrease speed of measurement only in part of measurement range, where correction is necessary.

Formulas (8) show that uncertainty of measurement increase at the end of the measurement range. In this place we measure two (or three) voltages, which have nearest values. In other parts of the measurement range \(U_0 > U_x\). Because of it the multiplicative uncertainty component is lower than additive one and sum uncertainty decreases.

To increase the speed of measurement and align the uncertainty of measurement on whole range of measurement it we proposed the algorithm, where the time of the voltage \(U_0\) measurement is directly proportional to the ratio of the voltages \(U_x/U_0\). This algorithm significantly increases the speed of measurement in main part of the measurement range.

3. Experimental results

Described above approach was used in logometric impedance meter MNS1200. MNS1200 was delivered to Siberian Institute of Metrology, to be used in wide frequency working inductance standard. Its appearance is shown on fig. 2.

Fig2. MNS1200 appearance

MNS1200 operates on range from DC to 1MHz.
Frequency set discreteness 2·10^{-5}
Capacitance range of measurement (F) 10^{-17}-10^{-5}
Resistance range of measurement (R) 10^6-10^{14}
Inductance range of measurement (H) 10^{-7}-10^{-10}
Dissipation factor \(\tg\delta\) (tg\(\phi\)) 10^{-6} – 1.0
Main uncertainty (ppm) 10
Sensitivity (ppm) 0.5
Inner standard instability (24 hours, ppm) ± 2

Fig3. Results of 1 Ohm resistive standard measurements during 24 hours
References


[13] Agilent PN 4294A; New Technologies for Accurate Impedance Measurement; (40 Hz to 110 MHz), Product Note.