Wide frequency range quadrature bridge comparator
Pont – comparateur a quadrator por une large bande de frequences

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Abstract. The quadrature bridge for comparison of the impedance standards in wide frequency and dynamic range was developed. The digital phase inversion of the operating signal is the single internal standard of this bridge. Technically bridge is based on the digital synthesizers of the sinusoidal signals, which are used as sources of quadrature voltages, supplying the standards to be compared. The appropriate algorithm of measurement eliminates the influence of the synthesizer’s uncertainty on the results of measurement. The bridge can compare the impedance of the standards on frequency range from units of Hz to units of kHz with uncertainty better than 1 ppm and resolution better than 0.03 ppm.

The quadrature bridge has a single internal standard in the form of digital phase inversion of the operating signal. The algorithm eliminates the effect of the uncertainty of the digital synthesizer on the measurement results. The bridge can compare the impedance of standards over a wide frequency range from units of Hz to units of kHz with an uncertainty better than 1 ppm and a resolution of 0.03 ppm.

Key Words / Mots Clés. impedance, comparison, quadrature, uncertainty, standard, digital synthesis, frequency range, algorithm, uncertainty elimination.

1. Introduction

The quadrature impedance comparison has a great importance for the fundamental and applied investigations. The investigations in this area have been expanding quickly since the 1960s. [1]. Up to present day, the quadrature impedance comparison was mostly used in fundamental investigations to trace the unit of impedance. The investigations in this area have been expanding quickly since the 1960s. [1]. Up to present day, the quadrature impedance comparison was mostly used in fundamental investigations to trace the unit of impedance.

2. Metrologic objectives of the project.

Project 2244 resulted in the creation of two automatic bridges-comparators and a set of intermediary standards, satisfying the following requirements:

4.1. The first comparator (autotransformer bridge) disseminates units of capacitance, resistance and inductance over a large dynamic range and reproduces the unit of inductance from capacitance and frequency over the inductive impedance range from 10 Ω to 1 MΩ. This comparator provides measurements with dissipation factor \( \tan \delta \) and tangent phase angle \( \tan \phi \) over the range of \( 10^{-7} \) to \( \pm (1-2) \). Its main uncertainty is better than of 0.5·10^{-6} to 2·10^{-6} and sensitivity is better than 10^{-8} [17].

4.2. The second comparator (quadrature bridge) provides \( \text{C} \leftrightarrow \text{R} \) transfer over the relatively narrow impedance dynamic range from 100 Ω to 100 kΩ with the standards, having \( \tan \delta \) close to zero (less than \( (1-2) \cdot 10^{-6} \)). Its main uncertainty is better than 1·10^{-6} and sensitivity is better than 3·10^{-6}.

4.3. Both comparators perform measurements automatically on 1.59 kHz and 1 kHz, use shared modules and are placed in the same case of weight less than 7 kg.

4.4. The set of thermostated intermediate standards contains standards, having cardinal values of the parameter (capacitance, resistance, and inductance) and special intermediate standards for \( \text{R} \leftrightarrow \text{C} \) and \( \text{C} \leftrightarrow \text{L} \) and sensitivity is better than 3·10^{-6}.

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transfer at 1 kHz. The ratios of the impedance of last standards to the cardinal values is equal to 1:0.4. These standards are tested by unit dissemination using first comparator. In this paper we describe the part of the results of this project, covering the development of the quadrature bridge-comparator

3. Principle of the bridge operation

To create the quadrature bridge we usually need three voltage generators $G_1$, $G_2$ and $G_3$, having voltages $(U)$, $(-U)$ and $(jU)$ respectively. Due to the different errors sources we write the real voltages of these generators in the form:

$$U_1 = U_0, U_2 = jU_0 (1 + \delta U), U_3 = - U_0 (1 + \delta i).$$

were: $\delta U$ is the error of the voltage ratio of the generators $G_1$ and $G_2$, $\delta i$ is the error of the inverter transfer coefficient. To create two counterphase voltages $U_1$ and $U_3$ in classic quadrature bridges [1] very accurate transformer divider usually are used. These bridges practically consist of two bridges. Every such bridge contains one pair of $R_1C_1$ $(R_2C_2)$ standards, one voltage source $U_2$ and common transformer divider. Both these bridges are simultaneously balanced. In the point of common bridges balance well known equation is valid:

$$\omega^2 R_1 R_2 C_1 C_2 = 1$$

Due to great efforts of many investigators today accuracy of the best (but handling) quadrature bridges achieve units of ppb. Of course, their frequency range and accuracy in this range are restricted by used transformer dividers.

In PTB, INRIM and other laboratories since the end of 70s investigators provided works, aimed to create the quadrature bridges for unlike impedance comparison on the base of the digital polyphase generators (synthesizers) [18,19]. These perspective works are provided today as well [21,22,23]. Last years a lot of works are devoted to use the synthesizers based on the Josephson array, but this very accurate solution is too expensive for commercial devises.

During the project 2244 implementation we used results, mentioned above, and improved them by calibration procedures, based on variational method [24]. First experimental tests of the quadrature bridge, developed in the frame of this project, were provided in PTB in 2005. These results were shown in [25]; the theoretical basis of these bridges was published in [26].

New quadrature bridge use only two digital generators $G_1$ and $G_3$, which generate the sinusoidal signals, having the same magnitude and shifted on 90°. The bridge compares impedances of one capacitive and one resistive standards.

Its simplified diagram is shown on Fig.1.
System software controls the whole process of comparison, processes the results of vector voltmeter measurements, and calculates measurement results.

The equivalent diagrams of the quadrature bridge on two stages of comparison are shown in Fig. 2a and Fig. 2b.

![Fig.2. Quadrature bridge equivalent diagrams](image)

Three main sources of the uncertainty influence on the result of measurement:
- ratio of two digital generators $U_1/U_2$;
- impedance of the connecting cables;
- instability of the bridge sensitivity.

To eliminate the influence of these uncertainty sources on the result of measurement we use the next approaches.
- Replacement method for unlike impedances on AC measurements [29] is used to eliminate the uncertainty $\delta U$ of generators voltage ratio.
- Algorithm of two unbalance signal measurements and their processing is used to eliminate influence of the “joke” $Z_j$ [30, 31].
- Variational method of the unbalance signal measurements [24] and processing is used to eliminate instability of the long-term bridge sensitivity.

Quadrature bridge operates by following algorithm:
1. The vector voltmeter $VV$ measures the quadrature components of the unbalance signal $U_{u3}$ in initial bridge state (see the Fig.2a).
2. Controlling system varies the magnitude or phase of one of the voltage generators on the well known value $\delta$, (it is preferable to change the phase of the generator $GU$, voltage). The vector voltmeter $VV$ measures the unbalance voltage $U_{n2}$.
3. Controlling system changes the state of the switcher $K_3$ and the vector voltmeter $VV$ measures the new unbalance signal $U_{n3}$.
4. Controlling system changes the state of the switchers $K_1$ and $K_2$, digitally inverts the phase of the generator $GU$, on opposite (see the Fig.2b) and the vector voltmeter $VV$ measures the new unbalance signal $U_{n4}$.

All this results of measurement are described by the next system of equations

$$\begin{align*}
U_0 &= \frac{U_0 - U_1(1 + \delta U)}{Z_R + Z_C} Z_R = U_{n1}; \\
U_0 &= \frac{U_0 - U_1(1 + 5\delta U)}{Z_R + Z_C} Z_R = U_{n2}; \\
U_0 &= \frac{U_0 - U_0(1 + \delta U)}{Z_R + Z_C} (Z_R + Z_I) = U_{n3}; \\
U_0 &= \frac{U_0 + U_0(1 + \delta U)}{Z_R + Z_C} Z_C = U_{n4}.
\end{align*}$$

where $U_0$ is the magnitude of the voltage generators, $Z_R$ and $Z_C$ are the impedances of the resistive and capacitive standards.

System solves the system of equation (3) accurately and finds the ratio of standards to be compared.

To analyze the influence of different factors on the result of measurement it is preferable to get the analytical solution of the system of equations (3). Such approximate solution is given by formula below:

$$\delta Z \approx \frac{\delta U}{2} \left( \frac{(U_{n4} - jU_{n3}) - (U_{n3} - U_{n1})}{U_{n2} - U_{n1}} \right)$$

Formula (4) gets the result of comparison with uncertainty better than $10^{-7}$ if $\delta U \leq \delta U_{\text{max}} = 3 \cdot 10^{-4}$, $Z_j/Z_z \leq 3 \cdot 10^{-4}$ and $\delta Z \leq \delta Z_{\text{max}} = 3 \cdot 10^{-4}$. It brings possibility to analyze the influence of all main factors on the result of measurement.

Formula (4) shows that the result of impedance ratio measurement depends on three differences: $(U_{n4} - U_{n1})$, $(U_{n3} - jU_{n3})$, and $U_{n2} - U_{n1}$ and variation $\delta w$.  

4. Analysis of uncertainty sources

4.1. Difference $(U_{n4} - U_{n1})$

Difference $(U_{n4} - U_{n1})$ is proportional to the deflection $\delta U$ of the generators voltages ratio from nominal value j. Usually $\delta U$ doesn’t exceed $(1-2)10^{-4}$ in frequency range up to units of kHz. The calculation by formula (4) eliminates the influence of this deflection on the result of measurement, but doesn’t eliminate the instability of this ratio during measurement process.

Instability of the generated voltages depends on two factors: instability of the used OpAmp gain and temperature instability of the used DAC.

To decrease the influence of the OpAmp gain on the voltage instability, voltage generators have two-cannel iterative structure (see Fig.3).

![Fig.3. Generator’s iterative structure.](image)

First channel consists of OpAmp $A_1$ and DAC$_1$. This cannel generates main part of the signal. The DAC$_2$ of the second cannel forms the low signal, proportional to the error of the first channel and, through OpAmp $A_2$, add it to the positive input of first channel. In this structure uncertainty, caused by limited values of the amplifiers gains can be estimated by formula

$$\delta g = \frac{1}{(K_1\beta_1)(K_2\beta_2)}$$

where $(K_1\beta_1)$ and $(K_2\beta_2)$ are open loop gains of the first and second channels. Let we will suppose that these values are the same for both channels and doesn’t exceed 0.02%. In this case $\delta g$ doesn’t exceed $4 \cdot 10^{-8}$ and its short term instability during measurement process doesn’t exceed units of $10^{-9}$.
We use in both voltage generators DAC with temperature coefficient better than 2 ppm/°C. To decrease the DAC temperature instability on the result of comparison, the DAC of the second channels are set into passive thermostat, so that DAC temperature instability during the measurement process (less than 20-30 sec.) doesn’t exceed 0.001-0.002 °C. In such way the generators voltage instability during the measurement doesn’t exceed 5-10 ppb.

4.2. Difference (U_{n3} − U_{n1})

Difference (U_{n3} − jU_{n1}) is proportional to the ratio $Z_j/Z_R$. The calculation by formula (4) eliminates the influence of this ratio on the result of measurement and ensures four terminal connection of the impedances to be compared in low potential part of comparator. But the calculation doesn’t eliminate instability of the ratio $Z_j/Z_R$ during the measurements. Usually $Z_j \leq 0.1$ Ohm (copper). Lowest impedance to be compared is 100 Ohm. Let the cable temperature instability during the measurement doesn’t exceed 0.01°C. Taking into account that copper temperature coefficient is $4 \times 10^{-5}$, uncertainty achieves $4 \times 10^{-5}$ on the lowest point (100 Ohm) of the impedance range of measurement.

4.3. Difference $U_{n2} − U_{n1}$

Difference $U_{n2} − U_{n1}$ is proportional to the variation $\delta_p$. Variation $\delta_p$ of the GU voltage is performed through digital phase variation on $\Delta \Phi_p$. Uncertainty of this variation is negligible and is restricted by the phase noise only.

4.4. Influence of the vector voltmeter accuracy

Analysis of the formula (4) shows that multiplicative and additive parts of theVV uncertainty doesn’t influence on the result of measurement. So that only VV noise and VV nonlinearity have to be taken into account.

Let we will describe the VV noise $\Delta U_{sn}$ and measured unbalance signals $U_{n1}$ as follow:

$$\Delta U_{sn} = a_{sn} + b_{sn}; U_{n1} = a_1 + j b_1;$$

where $a_{sn}$ and $b_{sn}$ are the quadrature components of the VV input noise, $a_1$ and $b_1$ are the quadrature components of the measured signals.

To get best accuracy, the magnitude of the variation has to satisfy inequality $\delta Z_{max} \leq \delta$, and the VV dynamic range $U_m$ has to be fully used ($U_m/2 \geq U_{n2} − U_{n1} \geq U_{n1}$). Suppose that $a_{sn} = b_{sn}$. Using these values, named assumptions and formula (4) we get approximate formulas to estimate the measurement uncertainty, caused by the input noise of the vector voltmeter:

$$\Delta (\delta Z)_n \approx 2 \sqrt{2} \frac{(1+j) a_{sn}}{U_m}; \delta (\delta Z)_n \approx 2 \sqrt{2} \frac{(1+j) a_{sn}}{U_m}.$$  (5)

Here $\Delta (\delta Z)_a$ and $\Delta (\delta Z)_n$ are the additive and $\delta (\delta Z)_a$ and $\delta (\delta Z)_n$ are the multiplicative components of the measurement uncertainty, caused by noise.

Formulas (5) show that uncertainty caused by voltmeter input noise can be almost three times bigger than the voltmeter conventional uncertainty $a_{sn}/U_m$ (this value for our voltmeter doesn’t exceed $10^{-5}$). Because of it uncertainty of the impedance comparison caused by voltmeter noise doesn’t exceed $3 \times 10^{-5}$.

4.5. Four terminal connection

To obtain four terminal connection in high potential part of the bridge we use the system of the master voltage generator and the slave current generator, shown on Fig. 4.

The slave current generator and the master voltage generator outputs are connected to the high potential current and potential ports of the standard. Input of the slave current generator is connected to current sensor (resistors $R_{c(i)}$). Current sensor enters into the feedback loop of the voltage generator. Every current generator generates the current $I_0$, which flows through impedances to be compared. The ratio of the residual current $\Delta I_r$, which flows through the output of the voltage generator, to the current $I_0$ is equal to: $\frac{\Delta I_r}{I_0} = R_c(s) Y_c(s)$, where $R_{c(i)} Y_c(s)$ is current generator’s GI open loop gain and $Y_{c(i)}$ is OpAmp $A_1$ and U/I converter transfer admittance.

Fig. 4. Four terminal connection

Assume cable impedance of $Z_c=0.1$ Ohm and the current generator’s GI open loop gain $R_c Y_c=10^6$ at upper frequency range. In this case uncertainty $\delta_{uc(s)}$, caused by cable impedance $Z_{c(i)}$ described by formula $\delta_{uc} = \frac{\Delta I_r}{I_0} \leq 1$ will not exceed $10^{-7}$ for standard impedance $Z = 100$ Ohm. Negative resistor $-R$ on Fig. 4 compensates the current, which is created by DAC feedback resistor $R$.

4.6. Four pair terminal connection

To get accurate impedance measurement and to exclude interferences, the currents, which flow through the central wire and screen of any connecting cable of measuring circuit has to be equal and has to have opposite direction [27, 28, 32].
To achieve in wide frequency range equality of the direct and back currents in the cables, which connect current GI and voltage GU generators with standards to be compared, in every of these units we use the special supply, which consists of pair of DC current generators and voltage stabilizers. In this case the screens of all cables are connected in appropriate points, as it is shown on Fig 5.

To measure correctly the unbalance signals the vector voltmeter is connected to the appropriate measuring circuit outputs and appropriate cable screen is grounded. In this case equivalent circuits (Fig2) correctly describe the measuring circuit (voltage drops between appropriate standard case and appropriate voltage generator ground is equal to zero). Equality of the currents in the cables centrals wire and screens we test using classical methods in higher part of the frequency range, where this question is most important. Tests have shown that relative inequality of currents in any cable doesn’t exceeds \((1-2) \times 10^{-4}\). Typically it decreases minimum uncertainty of measurement to units of ppb on upper frequency range.

4.7. Influence of the signal upper harmonics

The quadrature bridge compare unlike impedances, which in different ways depend on the frequency. The voltage generators contain not only operating frequency but also its higher harmonics. Because of it unbalance signal contains great level of higher harmonics, which create interference and lead to appropriate uncertainty. If generator voltage have relative harmonics level \(\delta_{gf}\) and vector voltmeter have relative selectivity \(\delta_{ef}\) estimated uncertainty \(\delta_f\) will not exceed \(\delta_f = \delta_{gf}\delta_{ef}\). Quadrature bridge uses vector voltmeter with \(\delta_{ef}\) lower than \((3-4) \times 10^{-5}\). Voltage generators have \(\delta_{gf} < 10^{-4}\). Therefore uncertainty satisfies \(\delta_f < (3 - 4) \times 10^{-9}\).

4.8. Influence of the digital phase inversion

To get bridge balance in both configurations – when resistive standard is connected to GU, and capacitive standard is connected to GU, or vice versa, we digitally reverse the phase of the generator GU. Accuracy of the digital phase reverse is restricted by phase noise only. This factor at highest frequencies of frequency range (units of kHz) doesn’t exceed \(10^{-9}\).

4. Conclusion

The developed automatic quadrature bridge potentially can compare impedances in wide frequency range. Upper frequency is limited mainly, by speed of used DAC and OpAmp and today is limited at units of kHz. Lower frequency of the frequency range is limited by time of measurement only. Uncertainty of measurement (as show uncertainty analysis) could be decreased to some parts of \(10^{-8}\). Experimental investigations, provided in PTB, have shown the difference in \(C \rightarrow R\) transfer lower than 0.4 ppm.

References


